Rigorous Function Calculi-IV

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Abstract

Almost all problems in applied mathematics, including the solution of various classes of differential equation, deal with spaces of real-valued functions on Euclidean domains in their formulation and solution. In this talk, I will describe the progress of a programme (Work Package 9 of CID) for developing implementations of various function spaces within the framework of computable analysis, and their implementation in the C++ package Ariadne [1, 2], together with some applications.

I will first briefly recall previous work on defining interfaces for the function types considered, covering (piecewise-)continuous, differentiable, analytic and elementary functions, measurable and integrable functions, and set-valued functions, including the C++ class definitions in Ariadne.

Next, I will discuss the relationship between *validated* and *effective* information about functions, with the latter allowing evaluation to arbitrary accuracy, and between *calling* a function, and a general *evaluation* operator. In particular, I will consider circumstances under which effective information can be used in a validated context.

Finally, I will discuss progress in the implementation of measurable functions and lower-measurable sets, and illustrate the use of the calculus of measurable functions to consider a simple example problem involving a stochastic dynamical system.

References

[1] Pieter Collins, Alberto Casagrande, Luca Geretti, Sanja Zivanovic-Gonzalez, Tiziano Villa, and et al. Ariadne: A C++ library for formal verification of cyber-physical systems, 2003-2020. `www.ariadne-cps.org`.

[2] Pieter Collins. Computable analysis with applications to dynamic systems. *Mathematical Structures in Computer Science*, 30(2):173–233, 2020.