Embeddability of graphs and Weihrauch degrees

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Abstract. In this talk, using the framework of Weihrauch reducibility, we aim to characterize the complexity of the problem S_H that asks whether a fixed graph H is isomorphic or not to an input graph G. The recursion and proof theoretic strength of this problem was considered in [HL96], where the authors analyzed the infinite versions of several problems coming from finite complexity theory. Recently, in [BHW21] the authors extended the work in [HL96] using the framework of Weihrauch reducibility. They considered H as either finite or as the linear graph with countably many vertices $(v_i)_{i\in\mathbb{N}}$ and edges $E = \{(v_i, v_{i+1} : i \in \mathbb{N}\}$. In the first case they showed that S_H is strongly Weihrauch equivalent to LPO (the problem that given in input $p \in 2^{\mathbb{N}}$ outputs 1 if p is not constantly zero) while in the second one they showed its equivalence with WF (the problem that given in input a tree $T \subseteq \mathbb{N}^{<\mathbb{N}}$ outputs 1 if T has no infinite branch). The second principle is actually much stronger than the first one, and the authors left open whether there is a graph H such that $\mathsf{LPO} <_{\mathsf{W}} \mathsf{S}_H <_{\mathsf{W}} \mathsf{WF}$: we provide an answer to this problem and we also consider the variant where subgraph isomorphism is substituted with subgraph embedding.

Keywords: Computable Analysis · Weihrauch Reducibility · Graph Embedding.

References

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