Quasi-Polish spaces as spaces of ideals

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Quasi-Polish spaces are a well-behaved class of countably based spaces which were originally studied to extend the descriptive set theory of Polish spaces beyond metrizable spaces [2], but were soon found to also have important applications in many other areas, such as locale theory [9, 4], logic [1], computable topology [12, 8, 10, 5], and the theory of valuations [3, 7].

In this talk, we will look at some connections between domain theory and a recent characterization of quasi-Polish spaces in terms of spaces of ideals of transitive relations on the natural numbers. This characterization was first shown in [8] as part of an investigation into the effective aspects of quasi-Polish spaces, and was further investigated in [5] and [6]. This generalizes the characterization of ω-continuous domains in terms of an abstract basis, but also preserves some important properties of an abstract basis. For example, in [5] we showed that M. Smyth's construction of powerdomains in [13] extends to this more general setting, and in [6] we used insights from Jones's work on probabilistic powerdomains in [11] to construct the space of valuations on quasi-Polish spaces as spaces of ideals. Although the powerspace constructions in [5, 6] focused on objects, we will show how they naturally extend to morphisms.

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