# Continuous Domain Theory in Logical Form

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*Dedicated to Samson Abramsky*
*on the occasion of his 60$^{th}$ birthday*

**Abstract.** In 1987 Samson Abramsky presented *Domain Theory in Logical Form* in the *Logic in Computer Science* conference. His contribution to the conference proceedings was honoured with the *Test-of-Time* award 20 years later. In this note I trace a particular line of research that arose from this landmark paper, one that was triggered by my collaboration with Samson on the article *Domain Theory* which was published as a chapter in the *Handbook of Logic in Computer Science* in 1994.

## 1 Personal Recollections

Without Samson, I would not be where I am today. In fact, I might not have chosen a career in computer science at all. Coming from a mathematics background I was introduced to continuous lattices by Klaus Keimel, and with their combination of order theory, topology and categorical structure, they seemed very interesting objects to study. It was only during my period as a post-doc working for Samson at Imperial College in 1989/90 that I became aware of their use in semantics. Ever since I have been fascinated by the interplay between mathematics and computer science, and how one subject enriches the other.

The time at Imperial was hugely educating for me and it had this quality primarily because of the productive and purposeful research atmosphere that Samson created. I believe in those days we went to the Senior Common Room for tea three times a day: in the morning, after lunch, and again in the afternoon. Usually, a large section of the Theory and Formal Methods group came along and it was our chance to talk about research problems that were on our mind. Samson was there most times and was happy to engage with any question that we brought up, and typically he would be able to point us to a relevant paper or result. We were forever astounded by his overview of the subject and his ability to quote to us not only theorems but also proofs.

In June 1992 Samson and I lectured at a summer school in Prague, organised by Jiří Adámek and Věra Trnková. My course was on domain theory, his on $\lambda$-calculus. One evening we had dinner together in one of this city's many charming restaurants and it was on that occasion that he invited me to become a co-author on a survey article on domain theory that was meant to form a chapter of the

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*Handbook of Logic in Computer Science*, edited jointly by him, Dov Gabbay and Tom Maibaum. I accepted but admittedly had little idea of what was involved; although I thought I knew a fair bit about the subject, it turned out that my knowledge was patchy and disorganised. I spent most of the year 1993 on this project, drafting chapter after chapter, sending them to Samson and receiving feedback, advice, criticism and encouragement back from him.

The article appeared in 1994 as [AJ94] and it has been pleasing to us how popular it has has been with researchers ever since.

## 2 The Handbook Article

Up to that point, domains were mostly conceived of as certain *algebraic directed-complete partial orders*, the most influential reference being Gordon Plotkin's *Pisa Lecture Notes*, [Plo81], which circulated widely in copied and re-copied form among researchers. The definition of an algebraic domain was first given by Dana Scott in 1969, [Sco69], in a note that also remained unpublished for many years, [Sco93], but Dana had moved quickly to the more "mathematically respectable" setting of complete lattices. Furthermore, he discovered that the notion of *algebraicity* could be replaced with a more general one, that of *continuity*. His *continuous lattices*, [Sco72], turned out to have many connections with mathematics and a period of fruitful collaboration between him and a group of mathematicians soon followed, culminating in the writing of the *Compendium of Continuous Lattices*, [GHK$^+$80].

When asked about the difference between "algebraic" and "continuous" structures in semantics, Dana's answer was that the latter were closed under an additional construction, that of forming retracts. In his view, this *ought to be* an advantage in setting up a denotational model. By 1993, this intuition was confirmed through the work on modelling probabilistic processes, [SD80, JP89], although continuous structures made their entrance through the real numbers, not through the need for general retractions.

Samson and I agreed that we would approach the subject of domains from the more general *continuous* angle. This suited me well because of my background in topology and functional analysis, and it seemed to offer a fresh perspective in the light of Gordon Plotkin's well-known treatment of the subject. It also forced us to engage with the "infinitary" dcpo structure of domains more deeply whereas many aspects of algebraic domains can be captured satisfactorily by the *poset* of compact elements.

The project went well, I think, and it was pleasing and sometimes surprising how easily and elegantly concepts known from the algebraic world could be generalised to the continuous setting. Early on I found that continuous domains could be generated from a more finitistic structure, which I dubbed *abstract bases*, but Samson pointed out that these had appeared in Mike Smyth's work before, [Smy77], under the name "R-structures". In any case, abstract bases were crucial for showing that it is possible to add operations (in the sense of universal algebra) in a free manner to continuous domains, and this established that the view of powerdomains as free constructions, first expounded in [HP79], worked here as well.

The final chapter of the article was devoted to Stone duality and Samson's *Domain Theory in Logical Form*, [Abr91b], which I will abbreviate to "DTLF" in this note. The general duality part was easy to do as we were able to import all our results from the *Compendium*, among them the beautiful characterisation of continuous domains given by Jimmie Lawson, [Law79], which says that they are precisely the Stone duals of completely distributive lattices.

Adapting Samson's work to the continuous setting, however, proved much more difficult. We didn't try for very long, as we ran out of time, so the version included in the Handbook chapter is for algebraic domains and the only "improvement" over [Abr91b] is that I renamed his "P predicate" to "C predicate." I was intrigued, however, and have spent a good part of my research time since then trying to extend Domain Theory in Logical Form to the continuous setting. Here I describe what I, together with collaborators, have found.

## 3 Domain Theory in Logical Form

At the heart of Samson's *Domain Theory in Logical Form* is the duality between *bounded distributive lattices* and *spectral spaces* discovered by Marshall Stone in the late 30s, [Sto37].[1] Three observations are key to its use in DTLF:

1. Most algebraic domains, when equipped with the Scott topology, are spectral spaces. In particular, this is true for Scott domains and the more encompassing class of bifinite domains.
2. Bounded distributive lattices are the Lindenbaum-Tarski algebras of negation-free propositional theories.
3. Constructions on algebraic domains have logical counterparts as free distributive lattice presentations.

To give an example of the last item, assume that the domain $D$ is the dual of the lattice $L$. Then the dual of the Plotkin powerdomain of $D$ can be presented as follows:

$$\begin{array}{lll}
\textbf{generators} & \{\Box a \mid a \in L\} \cup \{\Diamond a \mid a \in L\} & \\
\textbf{relations} & \Box(\bigwedge_{i\in I} a_i) = \bigwedge_{i\in I} \Box a_i & \Box 0 = 0 \\
 & \Diamond(\bigvee_{i\in I} a_i) = \bigvee_{i\in I} \Diamond a_i & \Diamond 1 = 1 \\
 & \Box(a \vee b) \leq \Box a \vee \Diamond b & \Box a \wedge \Diamond b \leq \Diamond(a \wedge b)
\end{array}$$

and the logical significance of the Plotkin powerdomain construction becomes immediately apparent.

[1] The paper remained far less well-known than his earlier [Sto36], possibly because mathematicians had no natural examples for spectral spaces and also, because the morphisms between them, now called *perfect maps*, seemed unnaturally restricted. Hilary Priestley's version of the duality, [Pri70], was much more successful.

The general setup of DTLF can be summarised in the following diagram:

$$\begin{tikzcd}
& \begin{array}{c}\text{program fragment}\\ P\colon \sigma\end{array} \arrow[dl, "\text{denotational interpretation}"'] \arrow[dr, "\text{logical interpretation}"] & \\
\begin{array}{c}\text{semantic space}\\ D_\sigma\end{array} \arrow[rr, leftrightarrow, "\text{Stone duality}"] & & \begin{array}{c}\text{program logic}\\ L_\sigma\end{array}
\end{tikzcd}$$

In this note, "semantic space" stands for algebraic or continuous domain but it could in fact be any type of structure employed to give a denotational meaning to programs. The "program logic" is typically propositional, and often enhanced with modal operators. Judgements are of the form $P\colon \sigma \models \phi$, where $\sigma$ is a type, $P$ is a program (fragment) of type $\sigma$, and $\phi$ is a formula in $L_\sigma$. Alternatively, the formulas in $L_\sigma$ can be used in "Hoare triples" $\{\phi\}\ P\ \{\psi\}$ with the usual interpretation. The fundamental idea of DTLF is that denotational and logical interpretation should determine each other *completely* via Stone duality.

As I said before, at the object level this works well for algebraic domains as long as one restricts to the bifinite ones. However, the topological maps that correspond to lattice homomorphisms are the *perfect* ones, i.e., those that are not only continuous but also reflect compact saturated sets.[2] Scott-continuous functions, the inevitable choice in domain theory, don't have that extra property.

Samson's solution to this puzzle was to distinguish between the "structural" category of domains, where the morphisms are embedding-projection pairs, and the Scott-continuous function space as a "type constructor." The fact is that the former *do* have nice counterparts under Stone duality, namely, lattice embeddings (injective lattice homomorphisms). One pay-off of this is that the somewhat technical *bilimit* construction of domains can dually be represented simply by a directed union of logical theories.

Extending this work to continuous domains requires a Stone duality that works for these spaces. At the time, the obvious choice was to move from lattices to *frames* which are known to be capable of representing all (sober) topological spaces, and to take advantage of the fact that continuous domains are indeed always sober in their Scott topology. The price to pay is that one is then working with an infinitary operation,[3] corresponding to the arbitrary union of open sets. There seemed to be no hope that this could be avoided as duals of ordinary (i.e., finite arity) algebraic structures always exhibit a zero-dimensional nature, and continuous spaces such as the real numbers just don't have that property. There was, however, Mike Smyth's then newly published work on a duality for stably compact spaces, [Smy92], which employed *proximity lattices* on the logical

[2] A set is *saturated* if it is upwards closed with respect to the specialisation order.

[3] More precisely, an operation of unbounded arity.

side. The difference to distributive lattices is that an idempotent relation $\prec$ is added to the algebraic structure, plus a number of axioms that link the two. In trying to understand Mike's paper, I played with a number of variations of these axioms, driven more by considerations of mathematical elegance than generality. It was Philipp Sünderhauf, then a PhD student at Darmstadt, who realised that one particularly pleasing set of axioms does indeed give a duality for all stably compact spaces:[4]

$$(\forall m \in M.\, m \prec a) \iff \bigvee M \prec a$$
$$(\forall m \in M.\, a \prec m) \iff a \prec \bigwedge M$$

In our paper [JS96] we called the resulting structure a *strong proximity lattice.*

**Theorem 1.** *The set* $\mathsf{spec}(L)$ *of round prime filters of a strong proximity lattice* $L$ *forms a stably compact space when equipped with the usual spectral topology generated by the sets* $\Phi(a) = \{F \in \mathsf{spec}(L) \mid a \in L\}$*. Conversely, given a stably compact space* $X$*, the sets* $(U, K)$ *with* $U$ *open,* $K$ *compact saturated, and* $U \subseteq K$ *form a strong proximity lattice, where are the lattice operations are the componentwise set-theoretic ones and the approximation relation is given by* $(U, K) \prec (U', K') \iff K \subseteq U'$*.*

Every distributive lattice carries a trivial proximity, namely the lattice order and so one sees that this theorem is a direct generalisation of that of Stone. However, many concepts from the classical case appear in a new light in the more general setting. Of particular importance to the story to be told here is the following: The unit map $\Phi$ of Stone duality maps a lattice element $a$ to the compact-open set $\Phi(a) = \{F \in \mathsf{spec}(L) \mid a \in F\}$. In Samson's setting this means that every set $\Phi(a)$ is of the form $\uparrow M$ with $M$ a finite set of compact elements. This is the link between domain logic and the concrete representation of algebraic domains as ideal completions of posets. On the other hand, the unit map of the generalised duality of Theorem 1 returns *pairs* $(U, K)$ where $U$ is an open set and $K$ compact saturated. If we view the elements of a strong proximity lattice as (equivalence classes of) propositional formulas, then this says that every formula $a$ has an *open* reading $[\![a]\!]^o$ and a *compact* reading $[\![a]\!]^c$ where furthermore $[\![a]\!]^o \subseteq [\![a]\!]^c$. The maps $[\![-]\!]^o$ and $[\![-]\!]^c$ are very well-behaved; they are lattice homomorphisms from $L$ to the frame of opens of $\mathsf{spec}L$ and the lattice $\mathcal{K}L$ of compact saturated sets, respectively. In fact, this is what sets the duality of strong proximity lattices apart from the one in [Smy92].

As in Samson's case, identifying the correct morphisms is not easy, and it has to be admitted that the paper [JS96] turns a blind eye to this problem. What we did provide was to define a Stone dual for continuous functions between stably compact spaces in the form of certain relations, modelled on Scott's *approximable mappings.*

[4] The definition of *stably compact space* is a bit involved and the interested reader is referred to [Jun04] or [GHK$^+$03] for a precise definition. As a first approximation, in a stably compact space the compact saturated sets behave exactly as compact sets do in Hausdorff spaces.

It was at this point that M. Andrew Moshier joined the effort, and he boldly changed our approximable mappings into relations between strong proximity lattices that resemble the internal approximation structure $\prec$. Furthermore, he realised that the axioms of strong proximity lattices look a lot more respectable when they are formulated as derivation rules for *sequents*, in the style of Gentzen's sequent calculus:

$$\frac{}{\bot \vdash}\,(L\bot) \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \bot}\,(R\bot)$$

$$\frac{\Gamma \vdash \Delta}{\top, \Gamma \vdash \Delta}\,(L\top) \qquad \frac{}{\vdash \top}\,(R\top)$$

$$\frac{\phi, \psi, \Gamma \vdash \Delta}{\phi \wedge \psi, \Gamma \vdash \Delta}\,(L\wedge) \qquad \frac{\Gamma \vdash \Delta, \phi \quad \Gamma \vdash \Delta, \psi}{\Gamma \vdash \Delta, \phi \wedge \psi}\,(R\wedge)$$

$$\frac{\phi, \Gamma \vdash \Delta \quad \psi, \Gamma \vdash \Delta}{\phi \vee \psi, \Gamma \vdash \Delta}\,(L\vee) \qquad \frac{\Gamma \vdash \Delta, \phi, \psi}{\Gamma \vdash \Delta, \phi \vee \psi}\,(R\vee)$$

$$\frac{\Gamma \vdash \Delta}{\Gamma', \Gamma \vdash \Delta, \Delta'}\,(\text{weakening})$$

(The comma between formulas on the left is meant to be read as a conjunction, and on the right as a disjunction. Double lines indicate that a rule can be read in both directions.)

The "forcing relation" $\vdash$ in these rules can be read alternatively as representing internal approximation $\prec$ or as a morphism between strong proximity lattices. A version of the cut-rule acts as composition. Importantly, the existence of an inverse to the cut-rule must be postulated to take account of the fact that $\prec$ is interpolative. We get the duality theorem:

**Theorem 2.** *The category of continuous sequent calculi and compatible consequence relations is dually equivalent to the category of stably compact spaces and closed relations.*

Without spelling out precisely the definitions of all the terms appearing in this theorem, perhaps the general flavour of the result can be appreciated: The duality is between a *logical* category of theories on the one hand, and a topological category with relations (rather than continuous maps), on the other.

Much of Samson's Domain Theory in Logical Form can be extended to this setting, and this was worked out by Mathias Kegelmann, [Keg99]. In particular, domain constructions can be given a "logical form". Mathias does this for product, coproduct, powerdomains, and the relation space; the bilimit construction is studied in [JKM01], and the example which originally motivated the move to continuous domains, the probabilistic powerdomain construction, is dealt with in [MJ02].

So far so good, but (at least) three questions remained:

1. How to capture the domain theoretic function space construction?
2. What are the "natural" morphisms of strong proximity lattices?
3. What is the role of the compact saturated interpretation $[\![-]\!]^c$ of propositions?

### 3.1 The Continuous Function Space Construction

Despite spending quite some time on this question, from the point of view of DTLF I consider it an open problem. We may take some consolation from the fact that the analogous problem in the algebraic setting caused Samson considerable difficulties, too. This is due to two facts. First, the category of algebraic domains is not closed under the continuous function space construction. As Smyth showed in his celebrated 1983 paper, [Smy83a], one has to restrict (at least) to *bifinite domains* if one wants to be certain that the function space between two domains is again algebraic. For Samson this meant that he had to impose additional axioms on his lattices to make sure that the Stone dual was indeed bifinite. Luckily, though, these additional axioms don't get much in the way in DTLF since one can always rely on the fact that, semantically, all constructions of interest return bifinite domains when applied to such structures.

Second, and more annoying, is the fact that a complete logical characterisation of the function space requires one to adopt the axiom

$$(a \to \bigvee_{i\in I} a'_i) = \bigvee_{i\in I}(a \to a'_i)$$

for all those formulas $a$ whose semantics is a *coprime* element in the lattice of open sets.[5] As a consequence, throughout DTLF one needs to keep track whether an element generated in one of the constructions has this property or not. Luckily, this can be done and the whole setup, though more complicated now, remains inductively definable.

Trying to transfer Samson's solution to the continuous world, there is good news and there is bad. The good news is that we know when we can expect a function space to be a continuous domain again; it happens when the inputs are *FS-domains*, [Jun90]. However, defining an analogue to Samson's coprimality predicate has so far exceeded this author's patience or ability. While it is clear that a coprime compact saturated set is one that is generated as an upper set by a single point, the condition for the corresponding open set would be that it is downward directed; in other words, it should be an open filter. Whether or not these two conditions can be tracked through all domain constructions, and especially the probabilistic powerdomain, I don't know.

Another problem makes its entrance at this point. Even if we knew how to formalise the Stone duals of FS-domains, we would not then be able to rely on the

[5] An element $a$ of a lattice is called *coprime* if it is contained in a finite union $\bigcup M$ of opens precisely if it is already contained in one of the $m \in M$, which is exactly what the axiom expresses.

fact that all our constructions preserve these conditions, contrary to the situation in classical DTLF. The issue is the probabilistic powerdomain construction for which it is not known whether it is closed on the class of FS-domains (nor on any other cartesian closed category of continuous domains), [JT98].

It turns out that an answer to the second question can be found by studying the third one, so this is how I will proceed now.

### 3.2 The Role of Compactness — First Interpretation

The interpretation of open sets in computation was expounded most clearly by Mike Smyth in his landmark paper [Smy83b]: They are exactly those properties which are *finitely observable.* This was a very fruitful view and in some ways DTLF is the logical extension of this insight. Compactness, on the other hand, while one of the basic notions of topology, is not that easy to interpret though by the time Samson and I wrote the Handbook chapter there were already a number of hints that it was a useful descriptional device: Gordon Plotkin had shown that the elements of his powerdomain could be characterised as convex compact[6] subsets of the given domain, and similar descriptions are available for the other two classical powerdomain constructions as well. He also formulated the intriguing "2/3 SFP Theorem" which says that two of the three conditions that characterise bifinite domains can be expressed by a compactness condition, namely, that the domain in question be stably compact in its Scott topology. Related to this is the role of compactness in the identification of maximal cartesian closed categories of continuous domains, [Jun90].

Since then Martín Escardó has shown [Esc04] that compactness is related to quantifiability, in the following sense: For $X$ some topological space one asks whether it is possible to establish whether a predicate, given as a continuous map from $X$ to $\mathbf{2}$ (Sierpiński space), holds for all elements of $X$. The answer is that this can be answered "continuously", that is, as a continuous map $\forall_X$ from $\mathbf{2}^X$ to $\mathbf{2}$ if and only if $X$ is compact. This is not just a theorem of topology but in fact a program can be written for $\forall_X$ provided $X$ is effectively given and the predicate to be tested is likewise given as a subroutine.

Another approach to compactness is to extend Steve Vickers's idea of a *topological system*, [Vic89], where elements of a "space" are related to elements of a frame by a relation $\Vdash$. The statement $x \Vdash a$ can then be read as "$x$ is an element of the open set $a$", or as "$x$ satisfies the observable property $a$", or as "$x$ is a model of the proposition $a$." In the given context one is tempted to replace "element" by "compact subset" and let $\prec$ play the role of $\Vdash$. The purely mathematical import of this has been explored by Olaf Klinke under the name *interaction algebra* in [Kli12].

All of the above, however, do not yet combine to produce a convincing story of why there is a compact interpretation of domain logic, nor what this compact interpretation represents, nor how it can be usefully employed in semantics. Indeed, what is missing is a serious case study of this approach in the same vein as

[6] With respect to the Lawson topology.

Samson's [Abr90, Abr91a]. An obvious candidate is to attempt a DTLF reconstruction of the striking result of Joseé Desharnais, Abbas Edalat and Prakash Panangaden, [DEP98, DEP02], about the completeness of a small and elegant Hennessy-Milner type logic for probabilistic processes.

### 3.3 The Role of Compactness — Second Interpretation

From the angle of Stone duality, some progress in extending and interpreting Theorem 1 has been made. The key insight is that on a stably compact space the complements of compact saturated sets form a topology, called the *co-compact topology*. In other words, stably compact spaces are *bitopological* structures and it is only because the two topologies in fact determine each other that this fact is not usually highlighted. Furthermore, a perfect map between such spaces is precisely one which is bicontinuous.

These observations motivate an alternative reading of the pairs $(U, K)$ in Theorem 1: The second component should be $X \setminus K$ and the condition $U \subseteq K$ should be read as $U \cap (X \setminus K) = \emptyset$. So the pair $(U, K)$ can be interpreted as a *partial predicate* in the sense of three-valued logic: it is (observably) true on $U$, (observably) false on $X \setminus K$ and undecided (or undecidable) everywhere else.

This turned out to be a fruitful starting point and in [JM06] Drew Moshier and I developed a duality theory for bitopological spaces analogous to the one for frames and topological spaces. More precisely, we define:

**Definition 1.** *A* d-frame *consists of two frames $L_+$ and $L_-$, together with two relations* $\mathsf{con}, \mathsf{tot} \subseteq L_+ \times L_-$. *Morphisms between d-frames are pairs $h_+, h_-$ of frame homomorphisms which preserve* $\mathsf{con}$ *and* $\mathsf{tot}$*.*

The following is now fairly straightforward:

**Theorem 3.** *There is a dual adjunction between the category of d-frames and the category of bitopological spaces.*

As is shown in [JM06], the duality of strong proximity lattices can be seen as a special case of Theorem 3, and the same is true for Stone's original dualities for Boolean algebras and distributive lattices, respectively. A particularly pleasing aspect is the fact that there is no doubt about the notion of a *d-frame homomorphism*; specialising them to the strong proximity lattice case one obtains what could rightly be called their natural morphisms. The fact that concretely they manifest themselves as pairs of relations perhaps explains why we were unable to identify them in [JS96].

With regards to DTLF, however, the bitopological or bilogical reading is yet to be fully justified. As we said before, the open interpretation of a proposition gives rise to the idea of observability or more precisely, semidecidability. An open that corresponds to the complement of the compact interpretation typically doesn't have that property and in some cases is very much *non-observable*. Why these complements form a topology, therefore, remains somewhat of a mystery — at least to this author.

## 4 Conclusions

It is probably fair to say that extending Samson's *Domain Theory in Logical Form* from algebraic domains to continuous ones has been a much harder task than we imagined in 1993, and it has forced us to examine very closely its various ingredients. While one could claim with some justification that the *multilingual sequent calculus* of Theorem 2 is the correct generalisation, some key questions remain open. What is more, making progress on these appears to depend on solving the long-standing problem of the behaviour of the probabilistic powerdomain construction on cartesian closed categories.

Let me end by expressing the hope that this summary of results and open problems will help to encourage researchers to study this fascinating and deep theory which Samson's work has opened up for us.

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